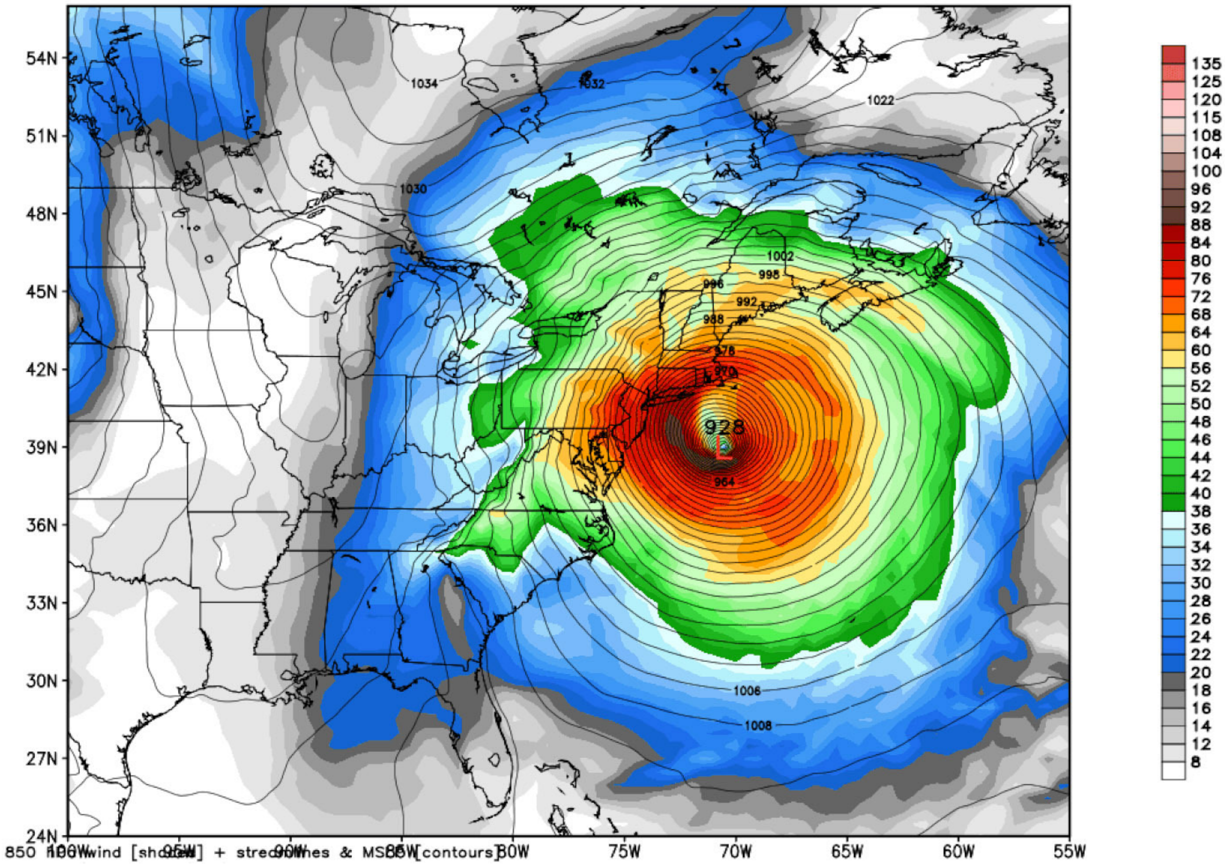
VIVIDSPARKS
CASE STUDY

POSIT™ for weather and climate models



We studied weather and climate models of low and medium complexity (shallow water model) we present benefits of POSITs compared to floats at 16bit. Our study focuses on geo-physical fluid simulations, the results are also meaningful and promising for reduced precision POSIT arithmetic in the wider field of computational fluid dynamics.

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Introduction

The forecast error of a weather forecast model has several origins: (i) Initial and boundary condition errors, which result from observations, data assimilation and external factors; (ii) model error, i.e. the difference between the mathematical model and the real world; (iii) discretization error resulting from a finite spatial and temporal resolution of the discretized equations and (iv) rounding errors with finite precision arithmetic. In general, the forecast error is dominated by (i-iii), depending on the forecast variable and the forecast lead time. In contrast, rounding errors are usually negligible with the IEEE 754 standard on 64bit double precision floating point numbers, which is still the standard for the majority of operational weather forecasts and in climate models.

Challenges

Weather and climate models provide predictions that are of great importance for society and economy. The Earth's climate system remains very difficult to predict even with the computational resources of the world's largest supercomputers, due to its complexity and non-linear dynamics that couple all features from the smallest time and length-scales to the largest. Reduced precision arithmetics is must for faster processing and communication between different elements of the computing architecture.

Approach

We will evaluate the different number formats (16bit half precision floats, 16bit posits with 0,1 or 2 exponent bits) when solving the shallow water equations. The shallow water equations result from a vertical integration of the Navier-Stokes equations under the assumption that horizontal length scales are much greater than vertical scales. This assumption holds for many features of the general circulation of atmosphere and ocean. The shallow water equations for the prognostic variables velocity *u = (u,v)* and sea surface elevation η are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + f\hat{\mathbf{z}} \times \mathbf{u} = -g\nabla\eta + \mathbf{D} + \mathbf{F} \qquad \text{a}$$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (\mathbf{u}h) = 0. \qquad \text{b}$$

For the atmosphere, η is interpreted as pressure. The shallow water system is forced with a zonal wind stress F. The dissipation term D removes energy on large scales (bottom friction) and on small scales (diffusion). The non-linear term (u ∇)u represents advection of momentum. The term $f\hat{z} \times u$ is the Coriolis force and $g\nabla\eta$ is the pressure gradient force, with g being the gravitational acceleration. Equation. b is the shallow water-variant of the continuity equation, ensuring conservation of mass. The domain is a zonally periodic rectangular channel of size 2000 km ×1000 km, with a meridional mountain ridge in the middle of the domain. The shallow water equations are discretized using 2nd order centred finite differences on an Arakawa C-grid with a grid spacing of Δ = 20 km (100x50 grid points) and the Runge-Kutta fourth order method is used for time integration. The advection terms are discretised using an energy and enstrophy conserving scheme.

Benefits

The solution to the shallow water equations includes vigorous turbulence that dominates a meandering zonal current. Using either float or POSIT arithmetic with 16bit the simulated fluid dynamics are very similar to a double precision reference: As shown in a snapshot of tracer concentration (Figure. 1) stirring and mixing can be well simulated with half precision floats and with 16bit POSITs (2 exponent bits). However, the half precision simulation (Figure. 1c) deviates much faster than the POSIT simulation (Figure. 1b) from the double precision reference (Figure. 1a). This provides a first evidence that the accumulated rounding errors with posits are smaller than with floats. To quantify differences between reduced precision arithmetics we perform model forecasts that compare rounding errors. The forecast error in the shallow water model is computed as root mean square error (RMSE).

Clearly the best forecast is obtained for POSIT arithmetic with 1 or 2 exponent bits (Figure. 2), with a small accumulation of rounding errors even for lead times of 100 days. The forecast error for 16 bit POSITs without exponent bit increases quickly (Figure. 2), especially for short forecast lead times, but a persistence forecast, i.e. assuming the initial conditions persist over time, is still worse (not shown). The forecast error of half precision floats is larger than the discretization error.

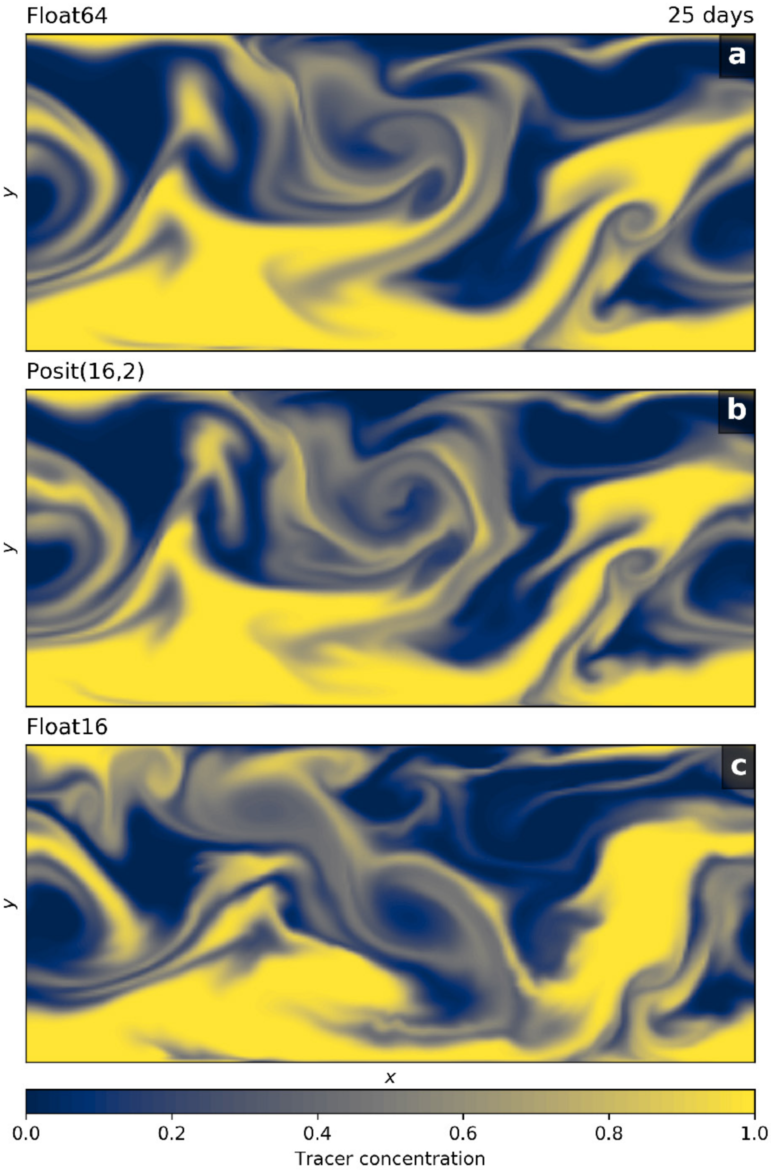

Figure 1: Snapshot of tracer concentration simulated by the shallow water model, based on (a) double precision floats and (b) POSIT arithmetic (16bit with 2 exponent bits) and (c) half precision floats. The tracer was injected uniformly in the left half of the domain 25 days before. This simulation was run at a resolution of $\Delta = 10$km (200x100 grid points).

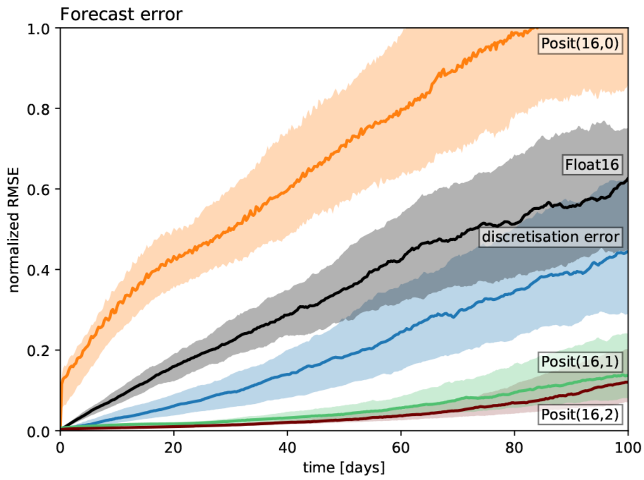

Figure 2: Forecast error measured as the root mean square error (RMSE) of sea surface height taking the double precision forecast as reference. The RMSE is normalised by a mean forecast error at very long lead times. Solid lines represent the median of 280 forecasts per number format. The shaded areas denote the interquartile range.

University Description

Oxford University, Department of Physics is a leader in physics nationally and internationally and their expertise to contribute to society's future through conducting cutting-edge research and by teaching and developing the careers of the next generation of physicists. They work on major facilities worldwide, develop the most advanced experimental techniques and the most sophisticated theoretical methods to investigate nature at every scale.